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Test of Scalar Meson Structure in ϕ Radiative Decays

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TEST OF SCALAR MESON STRUCTURE IN ϕ RADIATIVE DECAYS

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Abstract

We show that ϕ radiative decays into scalar mesons [$f_0(975)$,$a_0(980) \equiv S$] can provide important clues on the internal structures of these mesons. Radiative decay widths vary widely: $B.R. = 10^{-4} - 10^{-6}$ depending on the substructures ($q\bar{q}$, $qq\bar{q}\bar{q}$, $K\bar{K}$, glueball). Hence, we could discriminate among various models by measuring these widths at future ϕ factories. The understanding of these meson structures is valuable not only in hadron spectroscopy but also in nuclear physics in connection with the widely-used but little-understood σ meson. We also find that the decay $\phi \to S\gamma \to K^0\bar{K}^0\gamma$ is not strong enough to pose a significant background problem for studying CP violation via $\phi \to K^0\bar{K}^0$ at the ϕ factories.

1. Introduction to $\phi \to S\gamma$

This talk is based on the research done with F. E. Close and N. Isgur. For the details of our results presented in this paper, readers are suggested to read the joint paper in Ref. 1.

There are two major purposes for studying ϕ radiative decays. One is to understand scalar-meson [$f_0(975)$ and $a_0(980)$] structures and the other is to investigate a possible background problem for studies of CP violation at future ϕ factories.

Meson spectroscopy in the 1 GeV region has been well investigated and most mesons can be explained by naive $q\bar{q}$ models [2]. However, structures of the scalar mesons f_0 and a_0 (which we denote as S in this paper) are not well understood. There are a few "evidences" against identifying them with the $q\bar{q}$-type mesons. For example, if the strong-decay width of f_0 is calculated by assuming the $q\bar{q}$ structure, we obtain 500−1000 MeV width [3], which is in contradiction to the experimental one, 33.6 MeV [4]. Even if ambiguity of the hadronic matrix element is taken into account, the difference of an order of magnitude is too large. On the other hand, the scalar meson σ with the width of 500−1000 MeV has been widely used in nuclear physics [5]. Its

large width is essential for explaining isoscalar $\pi\pi$ phase shifts [6]. There are other evidences against the $q\bar{q}$ picture, for example, in studies of $\gamma\gamma$ couplings of S [7,8]. Considering these circumstances, we guess that the ordinary $^3P_0$ $q\bar{q}$ meson corresponds to the σ meson and the observed $f_0(975)$ is a more exotic meson, such as $qq\bar{q}\bar{q}$, $K\bar{K}$, or a glueball. The situation is also similar in the isovector-partner a_0-meson case [1]. It is important to understand the structures of these mesons not only in hadron spectroscopy but also in nuclear physics in connection with the σ meson.

There are ϕ factory proposals [9] at Frascati, KEK, Novosibirsk, and UCLA. We expect that some ϕ factories will be built in several years. One of our research purposes is to show that the scalar-meson structures could be investigated at the ϕ factories. Because the spin and parity of ϕ are 1^- and those of S are 0^+, the radiative decay $\phi \to S\gamma$ is an electric-dipole (E1) decay. The electric-dipole operator is given by $\sum e_i \vec{r}_i$, where e_i is the charge of a constituent and $\vec{r}_i$ is the vector distance from the center of mass. Therefore, its matrix element is very sensitive to the scalar-meson size. If S were the $q\bar{q}$ meson, charges should be confined in rather small space (about 0.5 fm size). On the other hand, if S is the $K\bar{K}$ molecule [6], the charges should be distributed in larger space (about 1.5 fm size). This is because scalar-meson masses are just below the $K\bar{K}$ threshold ($2m_{_{K^\pm}}$=987 MeV) and binding energies are small. There is an analogous case in nuclear physics. The deuteron exists slightly below the p-n threshold; hence it is a very loose bound state. From the above discussions, we can reasonably expect that charge distributions in the $q\bar{q}$ and $K\bar{K}$ cases are much different. The E1-decay width of $\phi \to S\gamma$ shall clearly reflect the difference. In this paper, we show our decay-width calculations based on the various models for the scalar mesons.

One of the major purposes for building the ϕ factories is to measure CP violation parameters accurately by using the decay $\phi \to K_S K_L$[10]. CP violation effects are small, so we should be careful in excluding possible backgrounds. An important background due to the radiative decay $\phi \to S\gamma \to K^0\bar{K}^0\gamma$ was recently pointed out [11,1,12]. Because the photon charge conjugation is negative, we have $K_S K_S$ or $K_L K_L$ instead of $K_S K_L$. If the branching ratio for the radiative decay is large [B.R.($\phi \to K^0\bar{K}^0\gamma$) $> 10^{-6}$], it is a significant background to the studies of CP violation. At this stage, there are several theoretical predictions for this branching ratio; however, they vary several orders of magnitude (B.R.=$10^{-5}-10^{-9}$) [11]. It is important to reach a theoretical agreement on the magnitude before we start the CP-violation experiments at the ϕ factories.

In section 2, the decay widths for $\phi \to S\gamma$ are estimated by using various models for the scalar mesons. In section 3, our investigation of the decay $\phi \to S\gamma$ through a $K\bar{K}$ loop is discussed by focusing on the $K\bar{K}$-molecule picture for S. Comments on the CP-background problem and on the OZI rule are given in section 4, and conclusions are in section 5.

2. Estimates of decay widths $\Gamma(\phi \to S\gamma)$ in various models for S

Assuming various models for S ($n\bar{n}$, $s\bar{s}$, $qq\bar{q}\bar{q}$, $K\bar{K}$, and a glueball), where n=u or d-quark, we calculate widths for the ϕ radiative decays. It should be noted that this type of investigation is still at the early stage, so that our calculations are naive order-of-magnitude estimates. Our purpose is simply to show that the decay widths are very dependent on the S structure; hence we could determine the structure by measuring them at the ϕ factories.

- $f_0 = s\bar{s}$ and $a_0 = (u\bar{u} - d\bar{d})/\sqrt{2}$

If f_0 is a $s\bar{s}$ bound state and a_0 is an ordinary $n\bar{n}$ state, we first estimate the decay width of $\phi \to f_0(s\bar{s})\gamma$ by using observed E1-decay widths. For example, the charmonium E1 decay $\chi_{c0} \to J/\psi\gamma$ is observed and the experimental width is 0.092 MeV. E1 decay widths are in general expressed as $\Gamma(E1) \sim e^2 R^2 E_\gamma^3$, where e is the charge factor, R is the hadron size, and E_γ is the emitted-photon energy. Taking into account these factors, we obtain

$$B.R.(\phi \to f_0\gamma) \approx \frac{1}{4.41} \cdot \frac{1}{3} \cdot (\frac{e_s}{e_c})^2 \cdot (\frac{R_s}{R_c})^2 \cdot (\frac{44}{320})^3 \cdot 0.092 \simeq 1 \times 10^{-5} \quad , \tag{1.1}$$

where 1/3 is a ϕ-spin factor and 4.41 MeV is the total ϕ decay width. We also used other E1 decays (e.g. $b_1(1235) \to \pi\gamma$) and obtained similar numerical results. The radiative decay into a_0 is an OZI violating process, so that the branching ratio should be significantly smaller than the one in Eq. (1.1):

$$R(a_0/f_0) \equiv \Gamma(\phi \to a_0\gamma)/\Gamma(\phi \to f_0\gamma) << 1 \quad . \tag{1.2}$$

- $f_0 = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $a_0 = (u\bar{u} - d\bar{d})/\sqrt{2}$

If both f_0 and a_0 are ordinary $n\bar{n}$ type mesons, both radiative decays are OZI-violating processes. Therefore, the decay widths are much smaller than the value in Eq. (1.1):

$$\Gamma(\phi \to f_0\gamma), \Gamma(\phi \to a_0\gamma) \lesssim 10^{-6} \quad . \tag{2}$$

Absolute values should be calculated by considering the $\phi - \omega$ mixing.

- $qq\bar{q}\bar{q}$

If the scalar mesons are four-quark bound states ($qq\bar{q}\bar{q}$) [13], we find that the decay-width ratio $R(a_0/f_0)$ provides us important information on the internal structure. The decay widths themselves are rather difficult to be estimated due to poorly-understood decays of four-quark bound states. In the following, we show that the ratio $R(a_0/f_0)$ varies significantly with the $qq\bar{q}\bar{q}$ structure. Because the electric-dipole operator is given by $\sum e_i \vec{r}_i$, we simply take into account constituent charges for evaluating the widths.

•• $qq\bar{q}\bar{q}$ [$K\bar{K}$-like "bag"=$(n\bar{s})(\bar{n}s)$]

If the f_0 and a_0 structures are $K\bar{K}$-like "bag", we denote $S = (u\bar{s})(\bar{u}s) \pm (d\bar{s})(\bar{d}s)$, where + is for f_0 and − for a_0. In this case, E1 matrix elements are roughly given by $M(E1) \sim (e_u + e_{\bar{s}}) \pm (e_d + e_{\bar{s}}) = 1$. Because the second term vanishes $(e_d + e_{\bar{s}} = 0)$, we obtain the same matrix elements for both decays. So, the decay-width ratio is

$$R(a_0/f_0) \approx 1 \quad . \tag{3}$$

•• $qq\bar{q}\bar{q}$ [$D\bar{D}$-like "bag"=$(ns)(\bar{n}\bar{s})$]

If the structures are diquark-antidiquark-like "bag", we denote $S = (us)(\bar{u}\bar{s}) \pm (ds)(\bar{d}\bar{s})$. The E1 matrix elements are given by $M(E1) \sim (e_u + e_s) \pm (e_d + e_s) = -1/3\ (f_0),\ 1\ (a_0)$ and the ratio is

$$R(a_0/f_0) \approx 9 \quad , \tag{4}$$

which is much different from the one in the $K\bar{K}$-like bag case. Therefore, we find that the decay-width ratio is also an important quantity for investigating the structures.

•• $qq\bar{q}\bar{q}$ [$\pi\eta$, $\eta\eta$-like "bag"= $(n\bar{n})(s\bar{s})$]

If the structures are $\pi\eta$ or $\eta\eta$-like "bag", we denote $S = (u\bar{u})(s\bar{s}) \pm (d\bar{d})(s\bar{s})$. The E1 matrix elements are given by $M(E1) \sim [(e_u + e_{\bar{u}}) - (e_s + e_{\bar{s}})] \pm [(e_d + e_{\bar{d}}) - (e_s + e_{\bar{s}})] = 0$, so that we cannot obtain the ratio by the above simple method. Much more detailed analyses are needed for the decay-width ratio and the widths themselves.

• $K\bar{K}$ molecule ("diffuse $K\bar{K}$")

The constituent-quark contents of the $qq\bar{q}\bar{q}$ and $K\bar{K}$ systems are identical, but their dynamical structures are very different. For example, it is clear that the deuteron should be regarded as the p-n bound state but not as a six-quark-bag-like bound state. The essential feature is whether the multiquark system is confined within a hadronic system with a radius of the order of $1/\Lambda_{QCD}$ or it is two identifiable color singlets spread over a region significantly greater than this. Decay-width calculations based on a $K\bar{K}$-molecule picture are discussed in section 3. The results indicate

$$B.R.(\phi \to f_0\gamma) \simeq 4 \times 10^{-5} \quad , \quad R(a_0/f_0) \approx 1 \quad . \tag{5}$$

This rate confirms our original expectation in the introduction that the E1-decay width should reflect the size of S; therefore, the width for the $K\bar{K}$ is larger than the ones for the $q\bar{q}$ models.

• f_0=glueball

If f_0 is a glueball, we estimate the quarkonium-glueball mixing by the observed $f_0 \to \pi\pi$ decay width and the calculated ${}^3P_0(q\bar{q}) \to \pi\pi$ width [3]: the mixing $\lesssim \Gamma(f_0 \to \pi\pi)/\Gamma({}^3P_0(q\bar{q}) \to \pi\pi) = 26/(500-1000) \lesssim 1/20$. Therefore, the decay width is $\Gamma(\phi \to f_0(glueball)\gamma) \lesssim \Gamma(\phi \to f_0(q\bar{q})\gamma)/20$. Using the results in Eqs. (1.1) and (2) for $\Gamma(\phi \to f_0(q\bar{q})\gamma)$, we obtain

$$B.R.(\phi \to f_0\gamma) \lesssim 10^{-6} \quad . \tag{6}$$

As we found in this section, not only the decay width $\Gamma(\phi \to f_o\gamma)$ but also the ratio $\Gamma(\phi \to a_0\gamma)/\Gamma(\phi \to f_0\gamma)$ is important for discriminating among the models. The obtained results are summarized in the following table.

Constitution		Ratio $\Gamma(\phi \to \gamma a_0)/\Gamma(\phi \to \gamma f_0)$	Absolute B.R.
$K\bar{K}$	molecule="diffuse $K\bar{K}$"	1	$a_0 \simeq f_0 \simeq 4 \times 10^{-5}$
$q^2\bar{q}^2$	$K\bar{K}$-like "bag"$= (n\bar{s})(\bar{n}s)$	1	$a_0, f_0 \lesssim 10^{-6}$? see Ref. 1
	$D\bar{D}$-like "bag"$= (ns)(\bar{n}\bar{s})$	9	
	$\pi\pi, \pi\eta$-like "bag"$= (n\bar{n})(s\bar{s})$	$-$	
${}^3P_0(q\bar{q})$	$f_0(n\bar{n})$, $a_0(n\bar{n})$	$-$	$a_0, f_0 \lesssim 10^{-6}$
	$f_0(s\bar{s})$, $a_0(n\bar{n})$	≈ 0	$f_0 \simeq 1 \times 10^{-5}$
glueball	f_0	$-$	$f_0 \lesssim 10^{-6}$

Table 1. Summary of possibilities

3. $\underline{\phi \to K\bar{K}\gamma \to S\gamma \text{ in a } K\bar{K} \text{ model for } S}$

We discuss our calculation of the decay width $\Gamma(\phi \to S\gamma)$ based on a $K\bar{K}$ model for S. There are existing literatures on the decay through a $K\bar{K}$ loop [11]. All of these calculations assume a pointlike coupling in the $SK\bar{K}$ coupling. However, it is not an appropriate description especially if the scalar meson S is the $K\bar{K}$ molecule. Because it is a loose bound state of K and $\bar{K}$, the keon-loop momentum cannot be an infinite quantity. The momentum should be restricted by the size of the bound system. In order to take into account of this effect, we introduce a momentum cutoff given by a momentum-space wave function for the $K\bar{K}$ molecule. For simplicity in our numerical analysis, we take the cutoff given by the dipole form factor: $\phi(|\vec{k}|) = 1/(1+\vec{k}^2/\mu^2)^2$, where the cutoff parameter μ is given by the $K\bar{K}$-molecule radius $R_{K\bar{K}}$ as $\mu = \sqrt{3}/(2R_{K\bar{K}})$. If we take the radius 1.2 fm, this cutoff is essentially the $K\bar{K}$-molecule wave function obtained by a Toronto group [7,6].

Once such momentum dependence is introduced, we have to be careful in current conservation or gauge invariance. By the minimal substitution $\phi(\vec{k}) \to \phi(\vec{k} - e\vec{A})$, we obtain a new current, so called "interaction current". The physics meaning of this current is as follows [14]. The finite-range form factor $\phi(k)$ at the $SK\bar{K}$ vertex means that K and $\bar{K}$ are annihilated into S with a finite distance r, which is controlled by the cutoff $\psi(r) = \int d^3k e^{i\vec{k}\cdot\vec{r}}\phi(|\vec{k}|)$. Therefore, current flows associated with this finite distance during $K\bar{K} \to S$ must be included in order to satisfy the current conservation. This is the physics meaning of the new current shown in Fig. 1d. Fortunately, photon energies in the ϕ radiative decays are about 40 MeV, which is considered to be a very-long-wavelength region in hadronic scale. Therefore, we simply expand $H_{SK\bar{K}} = g\phi(\vec{k} - e\vec{A})$ in the Taylor series and take the second term for obtaining the interaction current. Such a prescription should be good enough due to the soft photon [14]. In this way, we obtain the interaction current as

$$J^\mu_{int} = eg\phi'(|\vec{k}|)\hat{k}^\mu \quad , \tag{7}$$

where $\hat{k} = (0, \vec{k}/|\vec{k}|)$.

Now, we are in good preparation for calculating the decay width. Matrix elements in Fig. 1 are written as

$$M^\mu_a = -egg_\phi \int \frac{d^4k}{(2\pi)^4}\, \phi(|\vec{k}|)\, \frac{2\varepsilon^\mu_\phi}{D(k-q/2)D(k+q/2-p)} \tag{8.1}$$

$$M^\mu_b = +egg_\phi \int \frac{d^4k}{(2\pi)^4}\, \phi(|\vec{k}|)\, \frac{\varepsilon_\phi \cdot (2k+q-p)\ (2k)^\mu}{D(k+q/2)D(k-q/2)D(k+q/2-p)} \tag{8.2}$$

$$M^\mu_c = +egg_\phi \int \frac{d^4k}{(2\pi)^4}\, \phi(|\vec{k}|)\, \frac{\varepsilon_\phi \cdot (2k-q+p)\ (2k)^\mu}{D(k+q/2)D(k-q/2)D(k-q/2+p)} \tag{8.3}$$

$$M^\mu_d = +egg_\phi \int \frac{d^4k}{(2\pi)^4}\, \phi'(|\vec{k}|)\, \frac{\varepsilon_\phi \cdot (2k-p)\ \hat{k}^\mu}{D(k)D(k-p)} \tag{8.4}$$

where p, q, and k are the ϕ, photon, and keon-loop momenta respectively, and $D(k)$ is defined by $D(k) = k^2 - m^2_{_K}$. In the particular case where $\phi(|\vec{k}|) = 1$ and $\phi'(|\vec{k}|) = 0$, these reproduce the familiar field-theory expressions of Ref. 11. It is interesting to note the role that $\phi'(|\vec{k}|)$ plays in regularizing the infinite integral. Using these expressions, we can check explicitly the current conservation $\sum q \cdot M_j$=0. We define the matrix elements $\tilde{M}_j$ $(j = a - d)$ by $\tilde{M}_j = \varepsilon_\gamma \cdot M_j/(ie\varepsilon_\gamma \cdot \varepsilon_\phi)$ and the decay width is then calculated by

$$\Gamma(\phi \to S\gamma) \;=\; \frac{\alpha q}{3m_\phi^2}\,|\tilde{M}|^2 \quad , \quad \tilde{M} = \tilde{M}_a + \tilde{M}_b + \tilde{M}_c + \tilde{M}_d \quad . \tag{9}$$

Our numerical results for $\Gamma(\phi \to f_0\gamma)$ are shown in Fig. 2 as a function of the $K\bar{K}$-molecule radius. For the numerical evaluations, the coupling constants $g_\phi = 4.57$ and $g^2/4\pi = 0.6$ GeV^2 [11] are used. In the pointlike limit ($R_{K\bar{K}} \to 0$), we obtain the width 6.3×10^{-4} MeV, which exactly agrees with the value (marked by $\times$ in Fig. 2) obtained by using the analytical expressions in Ref. 11. However, if the $K\bar{K}$-molecule size 1.2 fm (the Toronto $K\bar{K}$ molecule) is used, we obtain the width 1.8×10^{-4} MeV, which is significantly smaller than the point-like result. Using the total ϕ width 4.41 MeV, we get the branching ratio $B.R.(\phi \to f_0\gamma) = 4 \times 10^{-5}$. In the $K\bar{K}$-type picture for a_0, the decay width for $\phi \to K\bar{K}\gamma \to a_0\gamma$ is roughly the same as the one for $\phi \to f_0\gamma$.

Fig. 1 $\phi \to K\bar{K}\gamma \to S\gamma$ processes.

Fig. 2 $\Gamma(\phi \to f_0\gamma)$ versus $R_{K\bar{K}}$.

4. Comments on a CP background problem and on the OZI rule

Because the photon charge conjugation is negative, $K_S K_S$ or $K_L K_L$ are produced in the ϕ radiative decays $\phi \to S\gamma \to K^0 \bar{K}^0 \gamma$. The ratio of CP violation parameters ε'/ε will be measured by using the decay $\phi \to K_S K_L$. If the branching ratios for the radiative decays are very large, it becomes a serious problem for measuring ε'/ε unless we find a method of excluding the radiative processes. Prior to our paper [1], there were a number of publications on this topic. However, it is rather surprizing to find that the branching ratio varies from 10^{-5} to 10^{-9}. This large fluctuation is due in part to errors and in part to differences in modelling [11]. (See the Brown-Close preprint in Ref. 11 for discussions on the previous calculations.) The resonant contribution to $\phi \to K^0 \bar{K}^0 \gamma$ is calculated by using the results in the previous sections and also the Breit-Wigner form for the S propagation:

$$\frac{d\Gamma(\phi \to S\gamma \to K^0 \bar{K}^0 \gamma)}{dp_s^2} = \frac{1}{(4\pi)^2} \sqrt{1 - \frac{4m_{_{K^0}}^2}{p_s^2}} \frac{g^2}{(p_s^2 - m_s^2)^2 + m_s^2 \Gamma_s^2} \Gamma(\phi \to S\gamma) \quad , \quad (10)$$

where p_s is the scalar-meson S momentum. We should note that there is a destructive interference between the f_0 and a_0 amplitudes. From the above equation and the results in sections 2 and 3, we obtain $B.R.(\phi \to K^0 \bar{K}^0 \gamma) \lesssim 10^{-7}$ [1,11]. This is too small to be a serious background to the CP-violation experiment.

Our investigations on the radiative decays suggest an interesting point on the OZI rule. If $f_0 = (u\bar{u} + d\bar{d})/\sqrt{2}$, the decay $\phi \to f_0\gamma$ should vanish in the lowest order and the (OZI-violating) $K\bar{K}$-loop contribution provides a small correction. If $f_0 = s\bar{s}$, we obtained the branching ratio for the "direct" process: $B.R.[\phi \to f_0(s\bar{s})\gamma] \approx 10^{-5}$. According to the OZI rule, this is supposed to be much larger than the OZI-violating decay $\phi \to K\bar{K}\gamma \to f_0\gamma$. If the $K\bar{K}$ system is diffuse ($R_{K\bar{K}}$ >2 fm), the loop process gives $B.R.(\phi \to f_0\gamma) < 10^{-5}$ from Fig. 2. In this case, the empirical OZI rule is valid. This is due to the poor spatial overlap between the $K\bar{K}$ system and the ϕ. Because the point-like calculation fails to take into account this confinement scale, the obtained rate is 10^{-4} and the OZI rule is invalid.

Next, assuming the $s\bar{s}$ for f_0, we consider the decay connected by a $q\bar{q}s\bar{s}$ intermediate state. There are two types of contributions from the $q\bar{q}s\bar{s}$ loops at the quark level. First, there are the diffuse contributions which can arise from hadronic loops corresponding to nearby thresholds, in this case from $K\bar{K}$. Then, there are short distance contributions. A realistic calculation of such contributions should include a large set of hadronic loops, and it was found that such hadronic loop contributions tend to cancel each other [15]. Therefore, the incompleteness of the cancellation of OZI-violating hadronic loops is due to nearby thresholds.

5. Conclusions

We investigated the ϕ radiative decays into the scalar mesons $f_0(975)$ and $a_0(980)$. We found that the decay widths vary widely: $B.R. = 10^{-4} - 10^{-6}$ depending on these meson substructures. Therefore, it should be possible to discriminate among various models ($q\bar{q}$, $qq\bar{q}\bar{q}$, $K\bar{K}$, glueball) by measuring these decay widths at future ϕ factories. However, our naive investigation is merely a starting point. Much detailed analyses are needed for the various substructure effects on the radiative decays.

We found that the radiative decay $\phi \to K^0\bar{K}^0\gamma$ is not a serious background to the CP-violation studies at the ϕ factories because the obtained branching ratio $B.R.(\phi \to K^0\bar{K}^0\gamma) \lesssim 10^{-7}$ is small.

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